Towards solving inverse optimal control in a bounded-error framework

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Abstract—In this paper, we apply inverse optimal control approaches in order to recover the cost function that can explain given observations, for a class of constrained optimization problems. The inverse optimal control was recently solved in an approximately optimal framework, meaning that the interest is in finding the proper criteria suitable for the system for which the decisions are approximately optimal. This method benefits of computational time efficiency and simplicity while solving the inverse optimal control problem, by simplifying the initial optimization problem into least square ones, easier than the first one. We focused on solving problems where systems and observations are both imperfect and uncertain. First, we test this method when working with uncertain observations, and results show that the method is sensitive to model uncertainties, encountering bias problems. Being inspired by the approximately optimal approach, we, secondly, use the idea given by this approach and propose a bounded-error approach to inverse optimal control; where all uncertainty and disturbances acting on observation or modeling are assumed bounded but otherwise unknown. A set membership algorithm is then proposed that compute bounds on the set of criteria that make the uncertain observations optimal. Then we show that the bounds computed for the criterion contains the actual solution.

I. INTRODUCTION

In a direct optimal control problem one should take into account the input and state trajectories that minimizes a known cost function. Whereas in an inverse optimal control problem, one should take into account the cost function which can explain given observations. The inverse optimal control is widely used for finding answers on how humans take their own decisions and how can we build robots with human like behavior.

A. Related work

1) Inverse optimal control: The inverse optimal control problem, a topic of study for many years now [1], has a variety of applications in different fields as networks, economics, control, robotics, automatics i.e. [2] solved an inverse optimal control problem by using prior knowledge of a control Lyapunov function and obtained a stabilization design for a rigid spacecraft. [3] studied inverse optimal control from an apprenticeship learning algorithms framework, that is not specific for the helicopters, and was able to create a first autonomous helicopter capable of flying as an human expert pilot. [4] analyzed trajectories of humans walking in crowds in order to transpose it on a nonholonomic robot, thus making the robot able to navigate through crowds. [5] solves the inverse optimal control by using the max-margin inverse reinforcement learning method, where the cost function that produces realistic trajectories needs to be recovered. [6] solves the inverse optimal control problem with the maximum planning method, by minimizing a cost function using an incremental sub-gradient method. This method was validated in a path planning for autonomous outdoor robots. [7] addressed the problem of inverse optimal control via inverse reinforcement learning for Markov decision processes based on linear programming. Using inverse reinforcement learning [8] developed a new method that enables us to recover the cost function, the policy and the value function. The advantage of this method is that once we have the value function we can explicitly write the cost function by using the Hamilton-Jacobi-Bellman equations. [9] used Thom’s transversality theory and proved that in order to give a proper solution to the inverse optimal control, three experiments having the same values of control are needed to recover the cost function, while two are not enough. They used this method so that HALE drones could decide by themselves as an expert human pilot. They studied the same problem in [10] where the same conclusions were drawn. [11] focused on finding the cost function, consisting on graph edges, by solving the shortest inverse path problem while given sets of observations of the shortest path.

[12] solved the inverse optimal control problem with a bi-level optimal method. They used observations from humans to recover the prior proper criteria. The method consists in solving two levels of optimal control, one to generate the optimal path and another to verify if the found criteria is fitting the observations. Once the criteria are found they implement the method on a humanoid robot and show that the robot is able to generate autonomous paths. [13], [14] used the same bi-level method for finding the criteria that can explain the arm pointing to a bar paradigm. In [14] is proved that this paradigm is better explained when minimizing a combined criteria. Moreover, the results from [13] support the previous idea and also show that even though we humans don’t have prior knowledge of the bar end-target we are restricting ourselves in the same region on the bar. [15] and [16] used the same method in order to explain and to better understand human arm motion in industrial screwing task. In these papers, they concluded that the criteria to be minimized, in this kind of human motion planning, is a combination of known criteria, mainly energy and geodesic expenditure.

[17] restrict their study to a convenient class of cost based on the observed trajectories in order to define the criterion that explains especially human locomotion data. In [18] they focused also on human locomotion, analyzing the class of optimal control that are convenient and that can explain

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the proposed problem. The idea of studying the class of optimal control comes from the fact that the observations of human locomotion are assumed to be the result of an optimization process. [19] makes use of the inverse optimal control to synthesize a controller based on a discrete-time stochastic control Lyapunov function. The advantage of solving the problem in this manner consists on the fact that the Hamiltonian-Jacobi-Bellman equations, which are a difficult task for this kind of systems, are not required to be solved. In contrast with [19], [20] propose a new inverse optimality design method based on the Hamilton-Jacobi-Bellman equations, where the cost function is constructed with the use of a linear function, mainly the Gaussian Radial Basic one.

[21] introduced a new method for recovering the cost function, based on approximately optimal condition. In fact, it is well known that when an optimal control problem is solved, in order to find the optimal solution, the Karush-Kuhn-Tucker conditions (denoted KKT) needs to be satisfied, meaning to be equal to zero. Contrary to this, the approximately optimal solved the problem by stating: given a set of observations, supposed to be optimal, it is sufficient to define the KKT residual functions and to solve them approximately, which means they are close to zero, in order to recover the minimized cost function. They introduced this method only for the class of convex optimization problems. They applied and validated their method in applications as consumer behavior, control and single commodity networks. [22], [23] and [24] were inspired by the inverse optimal control solved with the approximately optimal idea and used it in order to recover cost functions for deterministic discrete time systems [22], hybrid dynamical systems [23] and deterministic continuous time nonlinear systems.

The advantage of this method is its computational time efficiency and the ability of simplifying the initial constrained convex optimization problem into an unconstrained convex optimization one, easier than the first one. The studies that solved the inverse optimal control with the use of the approximately optimal idea assumed on the one hand that the observations are perfect measurements [24] and on the other hand that the observations are perfect while the system itself may be imperfect [22].

2) The bounded-error framework: Our interest is to work into a bounded-error framework [25] due to the fact that errors and uncertainties acting on the system and observations may have a barely known nature. We only assume the errors are bounded with known bounds, otherwise unknown. This approach was widely used and studied over the years, being applied to state and parameters estimation for nonlinear continuous-time systems [26], [27] or showing how to deal with the presence of uncertainty in the model and data, located within prior intervals [28].

B. Our contribution

The contribution of this paper are twofold. Building upon the Karush-Kuhn-Tucker conditions for optimality, we will first show how to deal with bounded uncertainty acting on the observations, and then show how to impute bounds for the set of feasible criteria consistent with the observation and the prior error bounds.

This paper is organized as follows. Sect. II recalls KKT conditions and the approximately inverse optimization approach. Sect. III gathers our new method for bounding the set of feasible criteria. Sect. IV illustrates our approach. Finally, we draw conclusions and we present what we are going to do next.

II. INVERSE OPTIMAL CONTROL

A. Problem formulation

We consider an optimization problem with constraints, in which a decision $x$ is made based on the optimization of a criterion subject to constraints:

$$\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad h_i(x) = 0, \quad i = 1, \ldots, m_1, \\
& \quad g_i(x) \leq 0, \quad i = 1, \ldots, m_2.
\end{align*}$$

where $x \in \mathbb{R}^n$ is the variable, $f(x)$ is the known criterion to be minimized, $g$ is the set of $m_1$ equality constraints such that $g(x) : \mathbb{R}^k \rightarrow \mathbb{R}^{m_1}$, while $h$ is the set of $m_2$ inequality constraints such that $h(x) : \mathbb{R}^k \rightarrow \mathbb{R}^{m_2}$.

B. KKT conditions for optimality

$x^*$ is the optimal solution of problem (1), if the following necessary and sufficient conditions are satisfied, namely if there exist the dual optimal $\lambda \in \mathbb{R}^n_+$ and $\nu \in \mathbb{R}^m$ that satisfy the KKT conditions:

$$\begin{align*}
\text{KKT}(x^*, \lambda, \nu) : \\
& \begin{cases}
    g_i(x^*) \leq 0, \quad \forall i = 1 : m_2 \\
    h_i(x^*) = 0, \quad \forall i = 1 : m_1 \\
    \nabla_x f(x^*) + \sum_{i=1}^{m_2} \lambda_i \nabla_x g_i(x^*) + \sum_{i=1}^{m_1} \nu_i \nabla_x h_i(x^*) = 0 \\
    \sum_{i=1}^{m_2} \lambda_i \nabla_x g_i(x^*) = 0 
\end{cases}
\end{align*}$$

where the first two conditions are primal feasibility, the third one is stationarity and the fourth condition is complementary slackness. The stationarity equation is also known as Lagrange duality that takes into account the constraints by augmenting the criteria with a weighted sum of the equality and inequality constraint functions. The complementary slackness implies:

$$\lambda_i > 0 \implies g_i(x^*) = 0 \quad \text{and} \quad g_i(x^*) < 0 \implies \lambda_i = 0 \quad \text{(3)}$$

This expression is known as the direct optimization problem and is used when one asks the optimal decision while optimizing a known criteria. In this study, we are interested on solving the inverse problem of (1), being known as the inverse optimal control problem.

Our purpose is to find the minimized criterion that can explain given set of observations composed of optimal decisions $x^*$.

To make the problem tractable, we will focus in this paper on the class of optimal control problems where the
sought criterion is written as a weighted sum, hence a linear combination of known criteria base functions. The criterion in (1) is now written as

$$f(x^*) = \sum_{i=0}^{k} c_i f_i(x^*).$$  \(\text{(4)}\)

where \(k\) is the number of pre-selected base functions and \(c = \{c_i\}\) is the unknown vector of weight values associated to each base functions \(f_i(x)\). When selecting the base functions, we usually have prior knowledge of the criteria, so a generic normalization method is used to normalize one criterion from the selected base functions. After normalization, equation (4) becomes:

$$f(x^*) = c_0 f_0(x^*) + \sum_{i=1}^{k} c_i f_i(x^*), \text{ where } c_0 = 1$$  \(\text{(5)}\)

Thanks to this parametrization, the KKT conditions (2) can now be rewritten in compact form introducing residuals [21], as follows

$$\text{KKT}(x^*, \lambda, \nu, c): \begin{cases} r_{\text{ineq}}(x^*) : g_i(x^*) \leq 0, \forall i = 1 : m_2 \\ r_{\text{eq}}(x^*) : h_i(x^*) = 0, \forall i = 1 : m_1 \\ r_i(c, \lambda, \nu, x^*) = 0 \\ r_{\text{comp}}(\lambda, x^*) = 0 \end{cases}$$  \(\text{(6)}\)

where the stationarity residual vector is

$$r_i(c, \lambda, \nu, x^*) = \nabla_x f_0(x^*) + \sum_{i=1}^{k} c_i \nabla_x f_i(x^*)$$

$$+ \sum_{i=1}^{m_2} \lambda_i \nabla_x g_i(x^*) + \sum_{i=1}^{m_1} \nu_i \nabla_x h_i(x^*)$$  \(\text{(7)}\)

and where the complementary slackness residual vector is

$$r_{\text{comp}}(\lambda, x^*) = \sum_{i=1}^{m_2} \lambda_i \nabla_x g_i(x^*)$$  \(\text{(8)}\)

Let first assume that primal feasibility is satisfied, i.e. \(r_{\text{ineq}}(x^*) \leq 0, \forall i = 1 : m_2\) and \(r_{\text{eq}}(x^*) = 0, \forall i = 1 : m_1\), then solving the inverse optimal problem resumes to finding the weight vector \(c \in \mathbb{R}^k_+\) and dual variables \(\lambda \in \mathbb{R}^{m_2}_+, \nu \in \mathbb{R}^{m_1}\) such that

$$(r_i(c, \lambda, \nu, x^*) = 0) \wedge (r_{\text{comp}}(\lambda, x^*) = 0)$$  \(\text{(9)}\)

**Inverse approximately optimal solution:** The idea underlying the approximately optimal solution resides in relaxing constraints (9) requiring only the residuals to be close to zero or merely minimized [21].

Observations \(x^*\) are now only approximately optimal, hence solving the inverse optimization problem boils down to:

$$\begin{array}{ll} \text{minimize} & \varphi(r_i(c, \lambda, \nu, x^*), r_{\text{comp}}(\lambda, x^*)) \\ \text{subject to} & \lambda_i \geq 0, \quad i = 1, \ldots, m_2. \\ & c_i \geq 0, \quad i = 1, \ldots, k \end{array}$$  \(\text{(10)}\)

where \(\varphi\) is a nonnegative convex penalty function that can be any norm \(\in \mathbb{R}^x \mathbb{R}^k\) or functions as Huber, dead-zone linear, log-barrier functions etc. Note that the solution is obtained from the stationarity and complementarity residuals, while the equality and inequality residuals are only used to check solution \(x^*\) feasibility.

Finally note that since the residual functions are linear with respect to the unknown variables \(c, \lambda\) and \(\nu\), the problem (10) is convex. Furthermore, if one solve (10) and find residual vectors \(r_i(c, \lambda, \nu, x^*)\) and \(r_{\text{comp}}(\lambda, x^*)\) that are close to zero, and if the measurements are primal feasible (\(r_{\text{ineq}}(x^*)\) and \(r_{\text{eq}}(x^*)\) close to zero), then the imputed weight vector \(c\), hence the imputed criterion is consistent with the observations.

### III. A BOUNDING APPROACH TO INVERSE OPTIMAL CONTROL

In this section, we introduce our bounding approach to inverse optimization.

In fact, the approximately optimal framework considered in section II assumes that all the uncertainty and errors acting on the system and the measurements can be summed up as additive disturbance on the residuals, hence explains why the residuals are not exactly zero. Here to the contrary, we will explicitly consider noise and disturbance on the observations, and also consider any modeling error.

In the sequel, we assume that the actual optimal variables are measured with bounded uncertainty, i.e. we assume that the optimal variables \(x^*\) are not exactly known but merely contained in a bounded set with known bounds. Here, we consider simple bounds on vector \(x^*\) components, i.e. bounded intervals \([x^*] = [\underline{x}^*, \overline{x}^*]\), where \(\underline{x}^*\) denotes the lower bounds of variable \(x^*\) and \(\overline{x}^*\) the upper bounds.

In this bounded error framework, solving (9) now becomes founding the weight vector \(c \in \mathbb{R}_+^k\) and the dual variables \(\lambda \in \mathbb{R}_{++}^{m_2}\) and \(\nu \in \mathbb{R}^{m_1}\) such that:

$$(\exists x \in [x^*], (r_i(c, \lambda, \nu, x) = 0) \wedge (r_{\text{comp}}(\lambda, x) = 0))$$  \(\text{(11)}\)

Because we are only mainly interested in imputing weight vector \(c\), inverse optimization boils down to computing the set of feasible vectors \(c\)

$$\mathbb{S} = \{c \in \mathbb{R}_+^k, \text{ s.t. } \exists \lambda \times \nu \times x \in \mathbb{R}_{++}^{m_2} \times \mathbb{R}^{m_1} \times [x^*], (r_i(c, \lambda, \nu, x) = 0) \wedge (r_{\text{comp}}(\lambda, x) = 0)\}$$  \(\text{(12)}\)

Noticing that the residuals are linear functions of the unknown variables vector \(c\), we can simplify the problem by computing only a bounding box \([\underline{c}, \overline{c}] = \text{AxisAlignedHull}(\mathbb{S})\) for \(\mathbb{S}\) [29], where the bounds are computed component-wise as follows. The upper bounds are obtained by solving \(k\) constrained maximization problems, whereas the lower bounds are obtained similarly by solving \(k\) constrained minimization problems:

$$\begin{array}{ll} \text{minimize} & \varphi(r_i(c, \lambda, \nu, x^*), r_{\text{comp}}(\lambda, x^*)) \\ \text{subject to} & \lambda_i \geq 0, \quad i = 1, \ldots, m_2. \\ & c_i \geq 0, \quad i = 1, \ldots, k \end{array}$$  \(\text{(13)}\)

$$\begin{array}{ll} \text{maximize} & \varphi(r_i(c, \lambda, \nu, x^*), r_{\text{comp}}(\lambda, x^*)) \\ \text{subject to} & \lambda_i \geq 0, \quad i = 1, \ldots, m_2. \\ & c_i \geq 0, \quad i = 1, \ldots, k \end{array}$$  \(\text{(14)}\)
∀j = 1... k, c_j ← max_{c, λ, ν, x} c_j subject to λ_i ≥ 0, i = 1,..., m_2 c_i ≥ 0, i = 1,..., k rs(c, λ, ν, x) = 0 r_{comp}(λ, x) = 0

Where both bounds satisfying
S ⊆ [c, z]

IV. APPLICATION TO SIMULATED DATA

In this section we test, using Matlab®, the two approaches presented earlier and validate the set-membership approach with simulated data generated for the unicycle robot type, model usually used [22], [24], [12] when validating new approaches for the inverse optimal control problem.

Firstly, we check the approximately approach sensitivity in the presence of noise in the observations, while trying to estimate one criterion. Secondly, we use the fact that this approach simplifies the initial problem into an unconstrained least square one and we propose a new approach in a bounded-error framework. We test our method by estimating first 2 criteria and after 3 criteria.

1) The unicycle model - trajectory generation: We consider the discrete model of the unicycle robot type:

\[
\begin{align*}
\dot{z}_1^{(i+1)} &= z_1^{(i)} + \tau_{c_3}^{(i)} \cos z_3^{(i)} \\
\dot{z}_2^{(i+1)} &= z_2^{(i)} + \tau_{c_4}^{(i)} \sin z_3^{(i)} \\
\dot{z}_3^{(i+1)} &= z_3^{(i)} + \tau_{c_5}^{(i)} \\
\dot{x}_v &= \dot{z}_3^{\text{start}} \\
x_{N-1} &= z_{\text{target}}
\end{align*}
\]

Where \( \tau \) is the sampling rate and \( i = 0 : N - 1 \) is the time step; \( z = [z_1(z_1), z_2(z_2), \ldots, z_5(z_5)]^T \) with \( z_1, z_2 \) the position, \( z_3 \) the orientation and \( z_4, z_5 \) the forward (linear) speed and angular speed respectively. The start and the end points are known.

The problem to be solved is an equality constrained optimization problem:

\[
\text{minimize} \quad 1 \over 2 \tau \sum_{i=0}^{N-1} (c_1^{(i)})^2 + (c_2^{(i)})^2
\]

subject to Eq. (15)

2) Inverse approximately optimal control approach: Based on equations formulated in Section II we apply approximately optimal control approach to (16) in order to recover the unknown weight \( c \) associated to the linear velocity. The problem to be solved is an equality constraint optimization one and the residual functions, presented in (9), associated to this application are:

\[
\begin{align*}
\eta_{eq} &= h_i(z)^*, i = 1,..., 3 \\
r_{i}(c, v) &= \nabla_{v} r_{eq}(z_{eq}^{(i)} + c \nabla_{v} r_{eq}(z_{eq}^{(i)}) + \sum_{i=1}^{3} v_i \nabla_{v} h_i(z)^*)
\end{align*}
\]

Where \( h_i(z)^* \) represent first three equality constraints presented in (16). In this approach the observations are supposed optimal therefore the \( \eta_{eq} \) are satisfied, whereas the \( r_s \), stationarity residuals, have the following form:

\[
r_{s}(c, v^{i+1}) = \begin{bmatrix}
-v_1^{i+1} \\
-v_2^{i+1}
\end{bmatrix}
\]

Finally, the solution is given by an unconstrained optimization problem, easier than the first one. Also the problem to be solved is a convex one based on the linearity with respect to the unknown parameters \([c, v^{i+1}]^T\):

\[
\text{minimize} \quad \|r_s(c, v^{i+1})\|^2
\]

The dimension of the residual is of \((5N, 1)\), where \( N \) is the number of time steps and \( z \in \mathbb{R}^5 \) This problem is solved using algorithms for unconstrained least-squares problems presented in [30].

Results: First we generate a trajectory using the unicycle model described in (16) with \( c \) chosen to be equal to 2. The observations on position, orientation, linear and angular velocities are used to solve the unconstrained least square optimization problem (19). Furthermore uniform distributed noise, with zero mean and three different values for the \( p \) variance is added to the observations and 300 trajectories are generated. We start by adding 1.5% of error to the positions and linear velocities, 3% error to the orientation and angular velocities and we generated 100 trajectories for this case. Afterward, the left 200 trajectories are generated by doubling two times the noise added to the observations as described previously and as presented in Fig. 1.

An unconstrained least square optimization problem is used to recover the \( c \) weight value for each generated trajectory. The results illustrate a well known fact, that is least square with noisy regressor yields biased results [31]. Fig. 2(b) shows that when noise variance is increased the method shows bias in finding the criteria that can explain the observations.

These results clearly show that the approximately inverse optimal approach is very sensitive to the presence of noise in the measurement. In order to reduce bias, one should add minima filter data or use more advanced estimation methods [31].

3) Inverse optimal control in a bounded-error framework: Two criteria case: In this subsection, we apply our method for the 300 generated trajectories, as in the previous case,
where the uncertainty vector of observation is defined as \( \mathbf{z} = [\mathbf{z}_1, \mathbf{z}_2] \).

Applying the methodology formulated in Section III for the unicycle model case (16), the problem to be solved is an equality constrained optimization one with the following KKT conditions associated:

\[
\begin{align*}
    h_i(z^*) & = 0 \ i = 1, \ldots, m \\
    \nabla_c \tau(z^*) + \nabla_c c(\tau(z^*)) + \sum_{i=1}^{N} v_i \nabla_c h_i(z^*) & = 0
\end{align*}
\]

Therefore the set of inverse optimal control solutions is defined as

\[
S: \{c \in \mathbb{R} \text{ s.t. } \exists \nu \in \mathbb{R}^3, \text{ (21) is satisfied}\}
\]

and its bounds can be computed using the min-max approaches (13).

**Results**

We applied this method using our simulated trajectory data while considering the three noise magnitudes.

**Table I**

| Noise \( \times 0.01 \) | Approx \( \min{[c_1 \ldots c_{100}]} \), \( \max{[c_1 \ldots c_{100}]} \) | Bounded \( |S| = |c, \tau| \) |
|-------------------------|---------------------------------|-----------------|
| 1.5 – 3                 | lower = 1.96; upper = 2.01     | \( |S| = [1.79, 2.25] \) |
| 3 – 6                   | lower = 1.89; upper = 1.98     | \( |S| = [1.56, 2.33] \) |
| 6 – 12                  | lower = 1.74; upper = 1.87     | \( |S| = [1.2, 2.32] \) |

Table I shows that our bounding approach takes correctly into account the presence of noise, though yielding larger uncertainty bounds, but still containing the true solution.

Fig. 2(a) shows that while increasing the magnitude of the added noise, our bounded-error approach exhibits larger uncertainty intervals but they all contain the true value. In all three cases the weight value \( c \) that can explain the observations is included into the found set of inverse optimal control solutions \( S \) returned by our method.

**Three criteria case:** In this subsection, we apply the bounded error approach in order to estimate the weights of 2 criteria, the linear velocity and the robots orientation respectively. First we generate data using (23) with \((c_1,c_2)\) chosen to be equal to \((5,1.5)\). As in previous case uniform distributed noise is added to the observations as follows: we first add \( \pm 0.4\% \) of error to the positions and linear velocities and \( \pm 0.2\% \) of error on orientation and angular velocities and we finish by adding \( \pm 3\% \) of error to the positions and linear velocities and \( \pm 1.5\% \) of error on orientation and angular velocities:

\[
\begin{align*}
    & \text{minimize } \frac{1}{2} \sum_{i=0}^{N-1} \left( c_1 (z_1^* - \nu_i)^2 + c_2 (z_2^* - \nu_i)^2 \right) \\
    & \text{subject to } \text{Eq. (15)}
\end{align*}
\]

As in the previous case, we apply the methodology from section III, define the associated KKT constraints and define the outer set of inverse optimal solutions as:

\[
S: \{c \in \mathbb{R}^2 \text{ s.t. } \exists \nu \in \mathbb{R}^3, \text{ (21) is satisfied}\}
\]

We calculate its bounds by solving (13) and finally we present the obtained outer set of solutions \( S \) in Table II. It can be seen that the true solution is within the obtained set of solutions. Also as shown in Fig. 3(b) and Fig. 3(d) within the set of solution we have true solutions but also we can find sets that are not solution to our problems. Nevertheless the objective of this study is to find the outer set of solutions in which we have the certainty that all true solutions are included. Fig. 3(a) and Fig. 3(c) are presenting a bisection over \( c_2 \) parameter.
that can give a good idea about the true set of solutions, represented in gray color.

Fig. 3. (a)-(c) The outer set of solutions represented by the green box and an approximation of the set of inverse optimal solutions, represented in gray. (b)-(d) Forward optimal control with parameter values from the set of solutions, represented with the red trajectory, and with parameter values from the outer set of solutions represented with the black trajectories.

V. CONCLUSION AND FUTURE WORK

In this study, we presented new ideas to solve the inverse optimal control problem that foster on ideas developed for approximately inverse optimization. Our method works in the bounded error framework explicitly taking into account any bounded uncertainty acting on the system and the measurements. Our method makes it possible to compute the outer set of criteria consistent with observations, a criteria base functions and prior error bounds on observations.

Future work will focus on alternative ways to a better characterization of the true solutions set or the inner set of inverse optimal control solutions.

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