Robust Fault Detection and Isolation
applied to Indoor Localization

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**Abstract:** In this paper, a passive method for robust fault detection and isolation is elaborated in the context of set-membership approach. The basic idea behind the proposed method is to compute a feasible set containing the states of a nonlinear system states using an interval constraints propagation method. A consistency test is performed using the $q$-relaxed intersection technique which consists in tolerating a given number $q$ of outliers from $m$ measurements. The robust feasible set, compatible with $m-q$ observations, is then obtained and faulty sets are used for the isolation procedure. An application, within the framework of robot/human localization, is given to verify the efficiency of the method.

**Keywords:** Fault detection and identification, set-membership functions, interval analysis, sensor network, data fusion, indoor localization.

1. INTRODUCTION

The principle of data outliers detection is based on residuals generation. Residuals are fault indicators obtained from two kinds of redundancies: hardware redundancy and analytical redundancy. The first one is based on the use of redundant sensors to measure a particular variable. Generally, a voting technique is applied to physical redundant systems. It consists in measuring the same variable by multiple sensors. In the set of measurements, the different signal is identified as default and used to isolate the faulty sensor. The use of physical redundancy in this manner is common in safety-critical plants such as aerospace vehicles and nuclear power systems. Hardware redundancy is easy to implement and quickly detects sensor faults but the major problems with this strategy are the extra equipment and maintenance cost and the additional space required. On the other hand, it is wise to measure the different values using the functional dependencies among the process variables rather than replicating each hardware device individually. This is the main idea of analytical redundancy which uses a set of algebraic or temporal relationships among the states, inputs and the output of the monitored process.

The aim of analytical redundancy in fault detection is to check consistency between a measured signal and its estimation. This requires an explicit mathematical model of the considered system. Any inconsistency expressed as residuals can be used for fault detection purposes. Hence, the residuals are necessary to reflect potential faults and inconsistencies. In fact, the residuals are close to zero when the monitored system is operating properly and should diverge from zero when a fault occurs in the system. However, due to the noise and disturbances the residuals may be non-zero valued in the absence of defaults, hence a robust fault detection and isolation is needed in order to provide valid residuals to the decision making stage. For this purpose, two methods have been developed to make the residuals maximally sensitive to faults and minimally insensitive to modelling errors and uncertainties: the active method and the passive method. In the former approach, optimal residuals are generated by several techniques such as unknown input observers, robust parity equation, $H_\infty$, etc. These methods are based on optimization in such a way that the effects of uncertainties are minimized with respect to the effects of faults on residuals. The purpose of the passive approach is to enable the uncertainties to be propagated into the range of residuals. This is the main idea of the set-membership approach: it aims to estimate the set of parameters or states in the “unknown but bounded error” paradigm. During the consistency check, when the measurements are not consistent with the set, a fault is assumed to have occurred.

Set-membership fault detection methods have been investigated by several researchers. In Bless et al. (2010), Planchon and Lunze (2008) the parameter uncertainty set was bounded by polytopes. Parallelotopic estimation was proposed in Ingimundarson et al. (2005) to describe the feasible parameter set within the framework of fault diagnosis. In Ingimundarson et al. (2008), Watkins and Yurkovich (1996) an orthotope containing the parameter vector was computed at every time instant using an update set-membership identification procedure. During the consistency test, a fault is detected when this orthotope is empty. A fault detection algorithm based on ellipsoids was proposed in Lesecq et al. (2003), Reppa and Tzes (2010). In Tzes and Le (1999) the feasible set of parameters was approximated by zonotopes for time-invariant systems. Assuming unknown but bounded perturbation and noises,
a state estimator based on interval analysis was presented in Marx et al. (2010), Raka and Combastel (2010), Adrot and Flaus (2008), Ploix and Adrot (2006), Drevelle and Bonnifait (2012). The passive approach for robust fault detection, based on set-membership approach, consists in evaluating the available observations which must be consistent with the interval of state estimation. This test is accomplished by an intersection between the observation set and the predicted one. During the checking of consistency between the state estimation and observation, an empty intersection may be obtained. That is why we propose in this work the use of a q-relaxed intersection which consists in tolerating a given number q of wrong measurements in order to obtain the feasible set. Such a technique is widely known to be suitable for outlier detection. Our purpose is to detect the faulty measurements using the q-relaxed intersection method.

Our work falls within the same framework of Drevelle and Bonnifait (2012). In this paper, we report in a different way the computation of the q-relaxed intersection and, through this process, the fault detection and identification approach. Then we apply this method to fault detection and isolation human/robot indoor localization using Pyroelectric Infra Red (PIR) sensor network.

This paper is organized as follows. In section 2, the problem is formulated within set membership framework and the fault diagnosis approach based on the q-relaxed intersection is introduced. Finally in section 3 and section 4 a practical application dealing with indoor localization is provided to show the performance of the passive robust diagnosis strategy presented in this paper.

2. INTERVAL BASED FAULT DIAGNOSIS

It is assumed in this paper that the system model can be described as follows:

\[
\begin{align*}
    x_{k+1} &= f_k(x_k) + \omega_k \\
    y_{k+1} &= h_k(x_{k+1}) + v_{k+1}
\end{align*}
\]  

(1)

where \(x_k \in \mathbb{R}^n\) (\(x_0 \in X_0\)) and \(y_{k+1} \in Y_k \subset \mathbb{R}^m\) are respectively the state and measurement vectors, \(f_k(\cdot)\) and \(h_k(\cdot)\) are nonlinear functions, \(\omega_k \in W_k \subset \mathbb{R}^n\) and \(v_{k+1} \in V_k \subset \mathbb{R}^m\) are respectively the process and measurement noise, \((X_0, W_k)\) and \((Y_k, V_k)\) are bounded sets of \(\mathbb{R}^n\) and \(\mathbb{R}^m\) respectively.

In the set-membership framework, the prediction step employs the previous state estimate to provide the predicted state using the dynamic model of the system. The predicted state set is defined as:

\[
X_k^+ = f_k(X_{k-1}^+) + W_k
\]  

(2)

During the correction step, the current measurements are used to update the predicted state:

\[
X_k^- = h_k^{-1}(Y_k)
\]  

(3)

The feasible set \(X_k\) depicting the estimated set is consistent with the prediction model, the actual data and the error bounds. Hence, \(X_k\) is defined by the intersection of sets \(X_k^+\) and \(X_k^-\):

\[
X_k = X_k^+ \cap X_k^-
\]  

(4)

2.1 Consistency test

During the feasible set computation, the intersection between the predicted set and the observations may be empty. This can be due to several causes such as inappropriate choices the initial set \(X_0\) or of the bounds for noise sets \(V_k\) and \(W_k\). The purpose of this paper is to accommodate the presence of faulty measurements in order to obtain a robust feasible set. For doing so, we propose the use of a q-relaxed intersection which is suitable for outliers detection, Jaulin (2009), Drevelle and Bonnifait (2010), Drevelle and Bonnifait (2011), described hereafter.

2.2 Relaxed set intersection

The robust feasible set may be computed considering the assumption below Jaulin (2009):

\textbf{Minimal Number of Outliers (MNO) assumption:}

Outliers may exist for the outputs but within any time window of l time steps, we never have more than q outliers.

A robust method can be used which is the q-relaxed intersection to overcome faulty data Jaulin (2009). This method consists in tolerating a given number q of outliers out of m measurements, and the solution set is then the set compatible with \(m - q\) measurements.

The q-relaxed intersection of m sets \(X_1, \ldots, X_m\) of \(\mathbb{R}^n\) is denoted by \(X^{(q)} = \bigcap_{i=1}^{m} X_i\). Thereby, if a measurement is inconsistent with the other measurements, it will be identified as an outlier, and is excluded from the solution set. This definition is illustrated by Fig. 1.

![Fig. 1. q-relaxed intersection of four sets for q ∈ {0, 1, 2}](image)

In Fig. 1, three cases are illustrated. In the first case, \(q = 0\), we obtain an empty intersection between 4 sets. In the second case we have a 1-relaxed solution which is obtained by relaxing 1 set \((q = 1)\) and intersecting the 3 remaining sets. A 2-relaxed solution is obtained when relaxing 2 sets \((q = 2)\) and intersecting the 2 by 2 remaining sets.

Computing the q-relaxed intersection of m sets has an exponential complexity. Using interval analysis the complexity becomes polynomial, Jaulin (2009). That is why we choose state estimation based on interval analysis.

2.3 Interval analysis

Interval analysis, Jaulin (2001), is based on the use of real intervals and interval vectors. An interval \([x]\) is a connected set of real numbers between a lower bound and an upper bound denoted \(\underline{x}\) and \(\overline{x}\) respectively.

\[
[x] = [\underline{x}, \overline{x}] = \{x \in \mathbb{R} | \underline{x} \leq x \leq \overline{x}\}
\]  

(5)
An interval vector, or a box, \([x]\) is a subset of \(\mathbb{R}^n\) that can be defined as the Cartesian product of \(n\) intervals. It is written as:

\[
[x] = [x_1] \times \ldots \times [x_n] = [\bar{x}_1, \underline{x}_1] \times \ldots \times [\bar{x}_n, \underline{x}_n]
\]  

(6)

The classical operations of real arithmetic and set-membership, namely addition (+), subtraction (-), multiplication (\(\times\)), division (/), intersection (\(\cap\)) and union (\(\cup\)) can be extended to intervals. For any operator, \(\circ \in \{+, -, \times, /\}\), we have:

\[
[x] \circ [y] = [x \circ y \in \mathbb{R} \cap [x, y] \in [y, y]]
\]  

(7)

Elementary functions such as \(\exp, \tan, \sin, \cos, \ldots\) can be extended to intervals:

\[
[f](x) = \{f(x) \cap [x]\}
\]  

(8)

### 2.4 Computing the \(q\)-relaxed intersection

The minimal number of outliers \(q\) is determined by GOMNE (Guaranteed Outlier Minimal Number Estimator), Jaulin et al. (1996). This algorithm computes a solution by increasing \(q\) beginning from 0. Once a non-empty solution during the intersection is achieved, \(q_{\text{min}}\) will be returned as the minimal number of outliers. The example below is provided to show how to compute the \(q\)-relaxed intersection between \(m = 3\) boxes Jaulin (2009).

- First, the corners of each box are projected on the \(x\) and \(y\) axis.
- Second, a paving is built using each two consecutive points on \(x\)-axis and two consecutive points on \(y\)-axis. The obtained paving consists of \((2m-1)^2 = 25\) boxes.
- Third, a midpoint is affected to each box.
- Then, an inclusion test is made. The test consists in searching the midpoint belonging to \(m - q\) boxes. Increasing \(q\), here, we obtain a non-empty solution for \(q = 1\). The box containing this midpoint is the \(q\)-relaxed solution.
- Finally, the set which does not contain the solution set is considered as an outlier unlike the others which are inliers.

**Fig. 2. Computing the \(q\)-relaxed intersection** Jaulin (2009)

The outliers detection using the \(q\)-relaxed intersection consists in checking if the solution set is consistent with all the measurements and the predicted set. Nevertheless, the identification procedure is more difficult. Indeed, we can obtain more than one box. In this case, the different boxes can not simultaneously contain all the measurements and the predicted set. In general, we can have three possible situations:

- (NO) No Outlier
- (OI) Outlier detected and Identified
- (OD) Outlier Detected but not identified

These different cases will be addressed in the next section.

### 2.5 Fault detection and identification

In this section, we adapt the principle of consistency check method of measurements with the solution set as in Drevelle and Bonnifait (2011). This is achieved when computing the \(q\)-relaxed intersection solution. In fact, the solution set is performed by an inclusion test made on midpoints of the paving as shown in Fig. 2. In Fig. 3, the solution set is the set containing the midpoint \(c_{13}\). This result is detailed in table 1. In each row, the "0" and "1" are the results of the inclusion test of each midpoint with each measurement. The last column "Sum" is obtained by adding the elements of the corresponding row. The midpoint of the solution set is the one having in the last column a value of \(m\) corresponding to the number of measurements. Otherwise, \(q\) is increased beginning from 1 and the midpoints of solutions are those having values of \(m - q\).

**Fig. 3. There is no outlier**

Table 1. Computation of the 0-relaxed solution. (We report only some of the midpoints due to the lack of space)

<table>
<thead>
<tr>
<th>Center</th>
<th>Measurement</th>
<th>([m1])</th>
<th>([m2])</th>
<th>([m3])</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(c_{11})</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(c_{12})</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

The midpoint of the solution set, obtained in table 1, will be used to draw the signature table which aims to detect and identify faulty measurements. In table 2, "0" and "1" indicate whether the measurement contains or not the midpoint of the solution set. The last column \([s]\) is obtained using a logical \(\text{AND}\) between the elements of the corresponding row. The last row is obtained using a logical \(\text{OR}\) between the elements of the corresponding column. In this case, table 2, no faulty measurement is detected.

**Table 2. Detection and identification with no faulty measurement**

<table>
<thead>
<tr>
<th>Center</th>
<th>([m1])</th>
<th>([m2])</th>
<th>([m3])</th>
<th>([s])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{13})</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

In Fig. 4 there is no midpoint in the paving since the number 3, which corresponds to the number of measurements, is not obtained in the last column. For \(q = 1\) the midpoint \(c_{17}\) corresponding to the 1-relaxed solution is determined by adding the measurements and obtaining the signature table. In this case, an outlier is detected and identified. A
Table 3. Computation of the 1-relaxed solution. (We report only some of the midpoints due to the lack of space.)

<table>
<thead>
<tr>
<th>Center</th>
<th>Measurement</th>
<th>[m1]</th>
<th>[m2]</th>
<th>[m3]</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_{17}</td>
<td></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Result</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

The outlier is detected and identified as \( m_2 \).

Table 4. Outlier detection and identification

| Center | Measurement | [m1] | [m2] | [m3] | | |
|--------|-------------|------|------|------|---|
| c_{17} |             | 1    | 0    | 1    | 0 |
|         | Result      | 1    | 0    | 1    | 0 |

Detection and identification

In Fig. 5, two midpoints, \( c_9 \) and \( c_{17} \), in the paving for \( q = 1 \) are obtained. The solution set is an union of the two non-connected boxes.

Table 5. Computation of the 1-relaxed solution. (We report only some of the midpoints due to the lack of space.)

| Center | Measurement | [m1] | [m2] | [m3] | | |
|--------|-------------|------|------|------|---|
| c_7    |             | 1    | 0    | 0    | 1 |
| c_9    |             | 1    | 0    | 1    | 2 |
| c_{17} |             | 1    | 0    | 1    | 2 |
| c_{25} |             | 0    | 0    | 0    | 0 |

The midpoints \( c_9 \) and \( c_{17} \) are used to obtain the signature table 6. In this case, outliers are detected according to last element of the column [s]. Nevertheless, no "0" is obtained in the last row corresponding to the columns of measurements. In this case, the outliers can not be identified since the measurements are not all consistent with the two boxes independently. Since \([m1]\) is the only column not containing zeros it is concluded that either \([m2]\) or \([m3]\) is the faulty measurement.

3. FAULT DIAGNOSIS APPLIED TO INDOOR LOCALIZATION

In this section, results of an experimental robot/human indoor localization are presented. The system model is described hereafter.

3.1 Mobility model

Several mobility models have been proposed in the literature to describe the motion of mobile entities, Camp et al. (2002). In our work, we aim to use a basic mobility model such that the algorithm remains general and applicable to as many situations as possible. A simple mobility model with minimum assumptions is the random walk model, Camp et al. (2002), Mourad et al. (2012), in which, an entity moves from its current location to a new one by randomly choosing a direction and speed. The speed, \( v \), and direction are both chosen from pre-defined ranges: \([v_{\text{min}}, v_{\text{max}}]\) and \([0, 2\pi]\) respectively. It assumes that only the maximal velocity \( v_{\text{max}} \) of entities is known. Between two time steps, the entity is able to move in any direction with a velocity less than \( v_{\text{max}} \). Hence, the mobility equation is formulated as follows:

\[
(x(t) - x(t - 1))^2 + (y(t) - y(t - 1))^2 \leq (\Delta t v_{\text{max}})^2 \tag{9}
\]

where \( x(t) \) and \( y(t) \) are the coordinates of the considered mobile entity at time \( t \). Knowing the point position for the considered mobile entity at time \( t - 1 \), the mobility constraint given (9) is a disk equation whose radius is \( \Delta t v_{\text{max}} \). This disk will be bounded by a box. When any additional informations on entities’ mobility could be added to refine the model. The problem is solved using interval analysis.

3.2 Observation model

The measurements consist in boxes representing the coverage sensing of PIR sensors. When a PIR is activated, the box representing its coverage is activated. At each time step, a list of boxes is provided. This list consists of the predicted box as given by the mobility model (10) and the measurement boxes. The location zone is obtained using the \( q \)-relaxed intersection of these boxes. A challenging task was to synchronize data coming from sensors. In our work, we acquire data at a one-second time step.

3.3 Proposed algorithm using interval analysis

The problem consists in finding the zone in which the person is located. In our method, every variable is replaced by a box. In particular, the point location coordinates \( p(t) = (x(t), y(t)) \) defined in (9) will be taken as a box: \(|p| \leq |x(t)| \times |y(t)| \). The two-dimensional box \(|p| \leq \Delta t v_{\text{max}} \) is computed through two phases:

- **Propagation phase**: Assume that \(|p| \leq \Delta t v_{\text{max}} \) is the sub-paving obtained at time \( t - 1 \). Computing the predicted

\[
|\hat{p}| \leq |\hat{p}| + (\Delta t v_{\text{max}}) |\Delta p| = |\hat{p}| + \Delta t v_{\text{max}} |\Delta p| - \Delta t v_{\text{max}} |\Delta p| = |\hat{p}| + \Delta t v_{\text{max}} |\Delta p|
\]

- **Detection phase**: Assume that \(|p| \leq \Delta t v_{\text{max}} \) is the sub-paving obtained at time \( t - 1 \). Computing the predicted

\[
|\hat{p}| \leq |\hat{p}| + (\Delta t v_{\text{max}}) |\Delta p| = |\hat{p}| + \Delta t v_{\text{max}} |\Delta p| - \Delta t v_{\text{max}} |\Delta p| = |\hat{p}| + \Delta t v_{\text{max}} |\Delta p|
\]
set \([p]_{estim}(t)\) may be achieved using (10). This box will be contracted using measurements during the correction phase.

**Correction phase.** During this phase, the predicted set \([p]_{estim}(t)\) will be refined using \(q\)-relaxed intersection with measurements. Every measurement will be considered as a constraint during the contraction. The propagation and the correction phase are illustrated by Fig. 6. The outliers detection using the \(q\)-relaxed intersection consists in checking if the solution set is consistent with all the measurements and the predicted set.

4. EXPERIMENTAL RESULTS

An actual motion for a person is performed in a Living Lab (GIS MADONAH) which is an actual apartment of 40 \(m^2\) inside the retirement home Bellevue at Bourges (France). To locate the person inside his room, the Living Lab GIS was equipped by five binary PIR sensors. The range of each sensor is \(6m \times 4m\). The repartition of sensors, as depicted in Fig. 7, allows us to track the person in the room. Generally, the sensors required in the smart-home are supposed to be non-wearable and non intrusive. Hence, the use of PIR detectors, in this work, as binary sensors. The detection area depends on the position choice of the sensor as shown in Fig. 8.

![Fig. 7. Sensors installation in the Living Lab](image)

Since the sensors range is too large, Fresnel lenses have been partially masked so that a zoning of the area is obtained. Each zone is covered in a discriminate way by a set of sensors, Fig. 9.

![Fig. 8. Technical characteristics of the PIR sensor](image)

The data collected by all these sensors are expanded and acquired using a KNX (Konnex) communication protocol for building automation on a computer in the technical room. Network integration in a KNX system is accomplished by a software installation tool based on a database (ETS Engineering Tool Software). At each time step, we acquire measurements during one second. An experimental collection of data, based on PIR sensors, will be provided to perform our algorithm.

Fig. 9. Repartition of sensors

The scenario consists in different activities made by a resident in his room such as crossing the bedroom or the hallway, immobility, sleeping and going to the bathroom. Our algorithm is able to provide an online location zone containing the true position. A part of the scenario is reported by Fig. 10.

![Fig. 10. Sensors installation in the Living Lab](image)

Fig. 11 shows the number of outliers detected at each time step. This result is performed by the outliers detection and identification method presented in section 3. Next, we will detail for particular time steps the result of outliers detection and isolation.

![Fig. 11. Number of detected outliers](image)

At time step \(t = 145s\), it can be noticed that no outliers are detected, Fig. 11.a. All measurements and the predicted set are consistent and the location zone, represented by the yellow box, is obtained. At time step \(t = 146s\) we have 3 measurements and a predicted set, Fig. 11.b. The solution set is performed by the two yellow boxes. In this case, one outlier \((q = 1)\) as we can notice from Fig. 11.b, is detected but can’t be identified. At time step \(t = 161s\) the outlier can be successfully detected and identified, Fig. 11.c.

5. CONCLUSION

In this paper, we presented a passive method for fault detection and identification based on the use of the \(q\)-relaxed intersection. This approach was used in the framework of robot/human indoor localization. An online location zone was performed using a sensor network based on PIR. Our algorithm was performed in a set-membership framework based on interval analysis. The method was evaluated through an experimental scenario in a Living Lab and it can be noticed that the approach is able to detect and identify faulty measurements while tracking and locating.
Fig. 10. Part 1 of the scenario: The resident is walking from the bathroom to the bed. The location zones (yellow) obtained by our algorithm contain real position at time steps 20s, 28s, 45s, 57s and 109s.

Fig. 11. Outliers detection: possible solutions. (a) No faulty measurement detected. (b) One outlier is detected but cannot be identified. (c) One outlier is detected and identified

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