Applications of Verified Methods for Solving Nonsmooth Initial Value Problems

Ekaterina Auer[1], Andreas Rauh[2]

[1]University of Duisburg-Essen, Germany, auer@inf.uni-due.de
[2]University of Rostock, Germany, andreas.rauh@uni-rostock.de

In many engineering applications, there is a need to choose mathematical models that depend on functions differentiable or continuous only piecewise. Such situations occur, for example, while modeling mechanical systems with friction, taking into account saturation effects in reaction kinetics and gas supply of fuel cells, or simply while trying to express naturally arising conditions such as nonpositivity of variables. The task becomes especially complicated if nonsmooth initial value problems (IVP) are considered. Here, even the definition of the solution depends on the application at hand [2]. Solving such problems is often additionally impeded by uncertainty in parameters. Besides, the solution is fairly sensitive to numerical errors. One possibility to deal with these difficulties is the use of verified methods both during the modeling and the simulation stages.

The development of verified methods for IVPs with nonsmooth right sides has got relatively few attention throughout the last three decades. To our knowledge, there exist no modern publicly available implementation at the moment. In [5], Rihm proposes a suitable definition and a method to enclose the solution to IVPs changing their right sides in dependence on a certain algebraic function. In [1, 3, 4], the authors propose algorithms for systems switching their representation according to graphs containing different ordinary differential equations as vertices and logical conditions as edges.

In this talk, we give an overview of the already existing methods for solving IVPs with nonsmooth right sides and propose a way of handling a certain practically relevant subclass of such problems. Our implementation uses the verified solver ValEncIA-IVP and a specialized template class for obtaining guaranteed enclosures of first order derivatives of nonsmooth functions. Next, we consider the example of a mechanical system with friction and hysteresis from [4] and compare the results of our approach to the results obtained with the help of the graph based one. We conclude by giving a perspective on applying such methods for modeling high temperature fuel cells and biomechanical systems.

References

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