Multi-Agent Low Range Sensing and the Among Constraint

SWIM 2011

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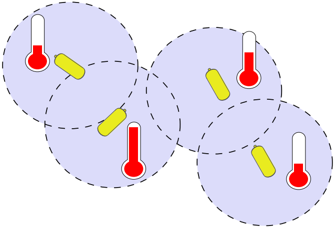
15 June 2011

Outline

Consider N agents using emitters/receivers



Signal :

- with low range
- omnidirectional
- satisfying a superimposition principle

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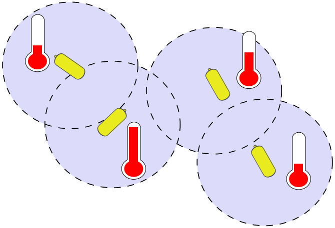

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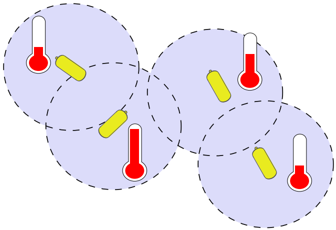

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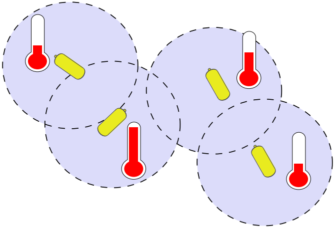

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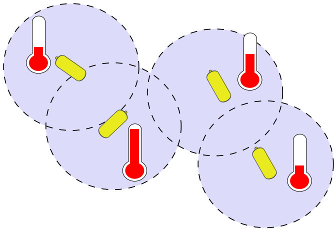

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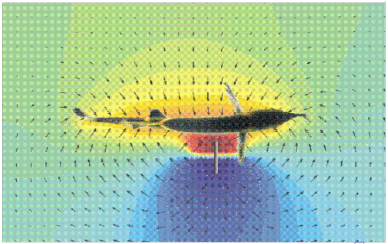
The example of the electric sense (*Angels project*)



Given the state (position) of n agents $x_1, \ldots, x_N$.
Evolution equation :

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{pmatrix} = f(x, u) = \begin{pmatrix} f_1(x_1, u) \\ \vdots \\ f_n(x_n, u) \end{pmatrix}$$

Observation equation (example) :

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = g(x, u) = \begin{pmatrix} \left(\sum_{i \neq 1} \frac{1}{\|x_i - x_1\|^k}\right) \\ \vdots \\ \left(\sum_{i \neq N} \frac{1}{\|x_i - x_n\|^k}\right) \end{pmatrix}$$

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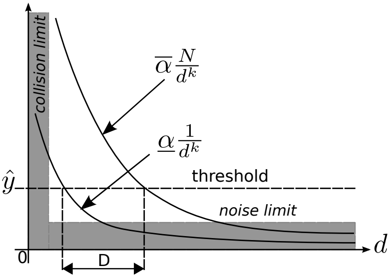

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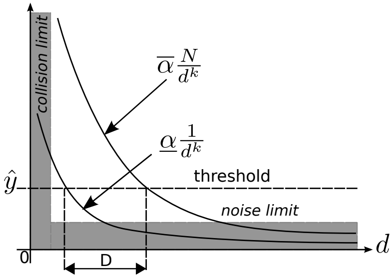

If $y_i > \hat{y}$ at least one agent is at distance $< \overline{D}$

If $y_i < \hat{y}$ no agent is at distance $< \underline{D}$

The Among constraint

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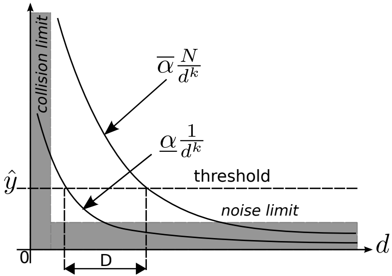

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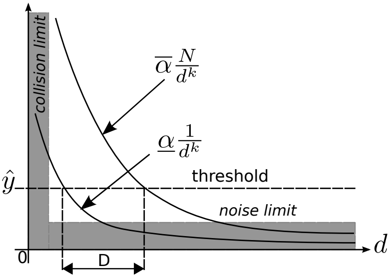

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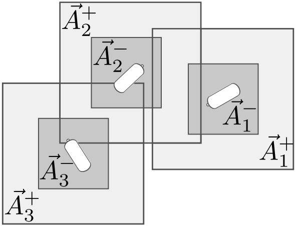
$\Rightarrow$ Very similar to Jaulin's approach (range only SLAM)

Inversion by hand (example)

Remark : for simplicity, assume 3 agents do not move. They are considered as **beacons**. Two others are "explorers".



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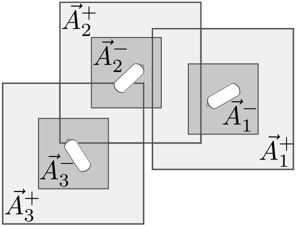

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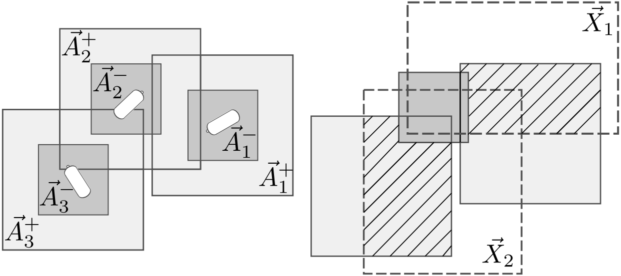

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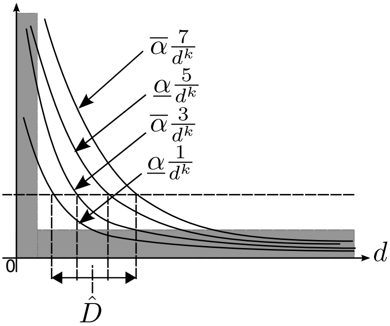
Now, the *y* function can be further discretized.



If no agent is less distant than $\hat{D}$ from the beacon then :
$y_i > \hat{y} \Longrightarrow$ there is between 3 and 7 agents at distance $< \overline{D}$

The Among constraint

This leads to the following generalization :

Generalization

Let :

$(\vec{x}_j)_{j \in J}$	be variables	(*agents positions*)
$\vec{V}$	be value domains	(*detection areas*)
K	a discrete inteval	(*bounds on the number of detected agents*)

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$$\texttt{among}(\ (\vec{x}_j)_{j \in J}, \vec{V}, K\ ) \quad \text{iff}$$

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In fact, we have here a conjunction of among (*as many as agents playing the role of beacons*).

Outline

Complexity

State of the art on the satisfiability of conjonctions of `among` :

- NP-Complete when V_i are not necessarily intervals
- But polynomial if they are all disjoints
- ... and when V_i are non-disjoints intervals ?
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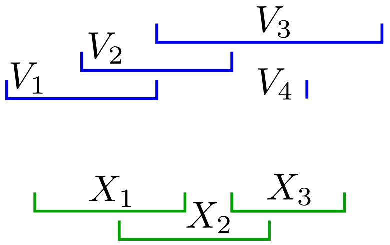

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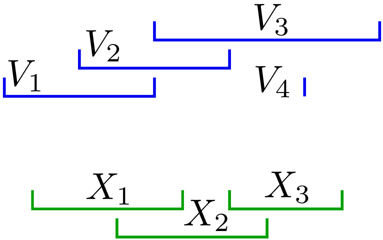

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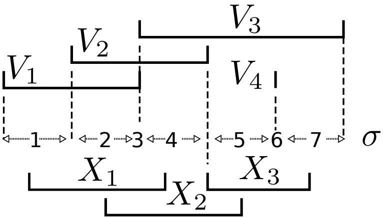
New Question : does exist $y_i \in \bigcup_{j=1}^{n} [x]_j$? such that

$$\begin{array}{c} \texttt{among}((y_1, \ldots, y_k), [x]_1, [1, k]) \\ \wedge \\ \texttt{among}((y_1, \ldots, y_k), [x]_n, [1, k]) \end{array} ?$$



We will count the number of variables that take their values inside the slice "s". Let $y_s = |j \in J, x_j = s|$.

- (1) is a linear program solvable in polynomial time
- (2) is polynomial (Regin's theorem)
- What about the conjunction of both ?

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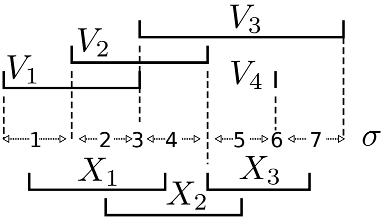

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A decomposition

$\texttt{among}(\ (\vec{x}_j)_{j \in J}, \vec{V}, K\ ) \quad \Longleftrightarrow$

$$\underline{k}_i \leq \sum_{s \in V_i} y_s \leq \overline{k}_i, \qquad \forall i \in I \qquad (1)$$

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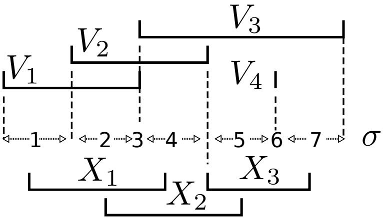

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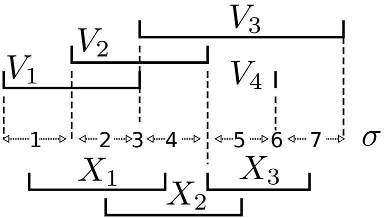

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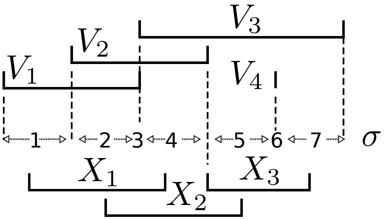

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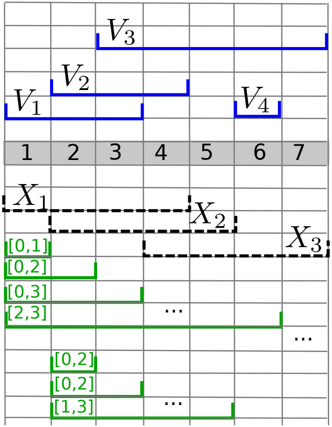
Key idea : "drop" (2) and "inflate" (1) :



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- We show that the maximal flow is n sing Hoffman's theorem

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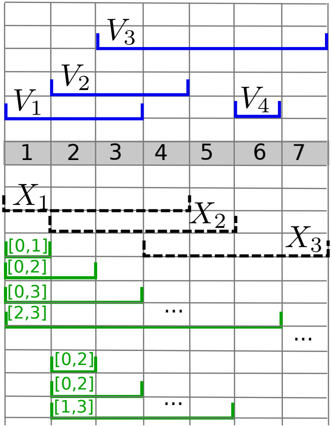

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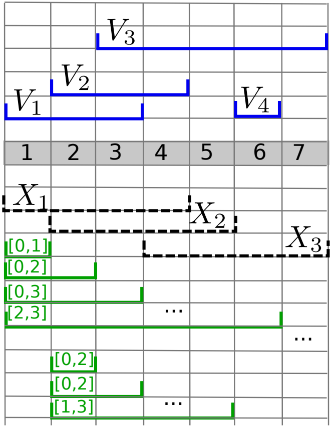

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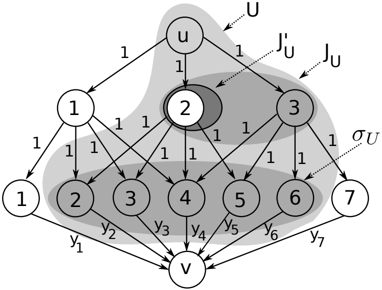

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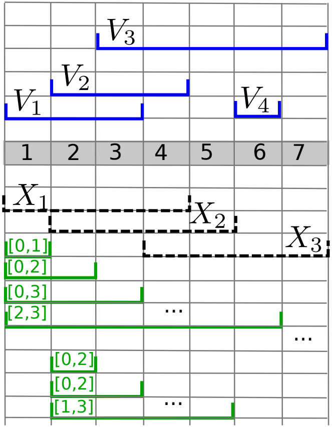

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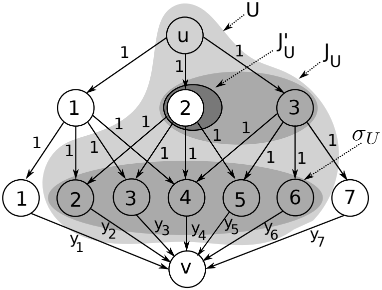

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- This makes unexpected connections with other fields of research (logistics)
- We have studied the complexity of a conjunction of `among`
- It is NP-hard with vector domains
- And polynomial in the one-dimensional case
- A filtering algorithm (not using Simplexe !) has been set up
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