Interval Analysis for Global Localization Problem using Range Sensors

Rémy GUYONNEAU, Sébastien LAGRANGE, Laurent HARDOUIN and Philippe LUCIDARME.

Laboratoire d’Ingénierie des Systèmes Automatisés (LISA),
University of Angers,
France.

June 13, 2011
Robot localization is an important issue of mobile robotics.

Robotics challenge CAROTTE\(^1\)

Simultaneous Localization And Mapping (SLAM) and Global Localization problems.

In this presentation a set membership approach will be considered to deal with the global localization problem.

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\(^1\)CArtographie par ROboT d’un TErritoire (Robot Land Mapping) organized by the french ANR (National Research Agency).
Outline

1. The Global Localization Context

2. The Proposed Method
   - The Static localization
   - The Global Localization Algorithm

3. The Simulator
1. The Global Localization Context

2. The Proposed Method
   - The Static localization
   - The Global Localization Algorithm

3. The Simulator
The Robot

The considered robot

We consider a mobile wheeled robot with a LIDAR\textsuperscript{a} sensor. Its pose is defined by $\mathbf{p} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$, with $(x, y)$ its localization and $\theta$ its orientation.

\textsuperscript{a}Light Detection And Ranging

The measurements

The sensor provides a set of $n$ measurements:

$$D = \begin{pmatrix} \mathbf{d}_0 = (d_{0x}, d_{0y}) \\ \vdots \\ \mathbf{d}_n = (d_{nx}, d_{ny}) \end{pmatrix}$$
## The Robot

![Diagram of robot and obstacle](image)

**Figure 1:** The robot's pose.

**Figure 2:** A measurement $d_i$. 

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The known environment $\mathbb{E} \in \mathbb{R}^2$ is discretized with a resolution $\delta_x$, $\delta_y$ and this lead to a grid $\mathbb{G}$ composed of $n \times m$ cells $(i, j)$. At each cell $(i, j)$ is associated $g_{i,j} \in \{0, 1\}$:

$$g_{i,j} = \begin{cases} 
1 & \text{if there is an obstacle in the cell } (i, j), \\
0 & \text{else.}
\end{cases}$$
The Objective

Hypotheses

→ Bounded error context,
→ Outliers are considered,

Figure 3: Measurements from the sensor.
The Objective

Figure 4: We have a grid map.

Hypotheses

- Bounded error context,
- Outliers are considered,

Figure 5: Measurements from the sensor.
The Objective

Figure 4: We have a grid map.

Hypotheses

→ Bounded error context,
→ Outliers are considered,

Figure 5: Measurements from the sensor.
The Objective

Figure 4: We have a grid map.

Hypotheses
- Bounded error context,
- Outliers are considered,

Figure 6: Initial domain.
The Objective

The diagram shows a 2D grid with a robot at the origin (0, 0) and range sensor measurements marked. The 3D representation on the right illustrates the angle θ ranging from 0 to 2π.
The Objective

The simulator provides a virtual environment for testing and validating localization algorithms. It allows for the simulation of various scenarios with different environmental conditions and obstacles. The objective is to accurately localize the robot in this simulated environment using range sensors. The results from the simulation can be used to evaluate the performance and robustness of the proposed method.

The diagram on the left shows a two-dimensional representation of the environment with a grid layout, representing obstacles and open spaces. The orange dots indicate the range sensors' measurements from the robot's perspective. The right diagram illustrates a three-dimensional representation, emphasizing the spatial orientation of the robot's position and orientation in a virtual space, with axes labeled as $x$, $y$, and $\theta$.
The Objective

The robot is depicted in a two-dimensional grid with sensors indicating its possible locations. The robot's objective is to determine its exact position within the grid. The grid is marked with coordinates (0, 0) and has a range sensor radius indicated by the red lines emanating from the robot. The 3D representation on the right side of the diagram visualizes the range sensor data in a three-dimensional space, with the robot's orientation (θ) and position (x, y) estimated within the environment.
The Objective

The diagram illustrates a grid-based representation of a global localization problem. The grid is used to map the possible locations and orientations of a robot within a defined space. The coordinates $(x, y)$ and orientation $\theta$ are indicated, with the origin $(0, 0)$ marked. The grid cells represent potential positions, with some cells highlighted to indicate possible locations based on sensor inputs. The objective is to accurately localize the robot within this space using interval analysis and range sensors.
The robot is localized

All the measurements fit with the map.
The robot is localized

Except for one outlier.
The Proposed Method

1. The Global Localization Context

2. The Proposed Method
   - The Static localization
   - The Global Localization Algorithm

3. The Simulator
Let \( \mathbf{p} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \) be an initial domain that encloses the robot’s pose \( \mathbf{p} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \) and \( \mathbf{D} = \begin{bmatrix} d_0 \\ \vdots \\ d_n \end{bmatrix} \) a set of \( n \) measurements.
The Static localization

The Robot-Measurement Distance

$$||d_i||^2 = (x - w_{ix})^2 + (y - w_{iy})^2.$$
The Measurement-Measurement Distance

\[ ||d_i - d_j||^2 = (w_{ix} - w_{jx})^2 + (w_{iy} - w_{jy})^2. \]
The coordinates \((w_{ix}, w_{iy})\) of an obstacle in the map are defined by:

\[
\begin{pmatrix}
w_{ix} \\
w_{iy}
\end{pmatrix} = \begin{pmatrix}
cos(\theta) & sin(\theta) \\
-sin(\theta) & cos(\theta)
\end{pmatrix} \begin{pmatrix}
d_{ix} \\
d_{iy}
\end{pmatrix} + \begin{pmatrix}
x \\
y
\end{pmatrix}
\]
The Static localization

The Global Localization Context
The Proposed Method
The Simulator

The Static localization

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The Static localization
The Static localization
We define $c_G$ the constraint which says that the measurement has to be consistent with the map.
The Static localization
The Static localization

We define $C_G$ the contractor of the constraint $c_G$. 

The map contractor.
The Static localization
Considered Constraint Satisfaction Problem

\[ \mathcal{V} = \{x, y, \theta, d_i = (d_{ix}, d_{iy})^T, w_{ix}, w_{iy} \text{ with } i = 1, \cdots, n\}, \]

\[ \mathcal{D} = \{ \]
\[ [x] = [-\infty, +\infty], [y] = [-\infty, +\infty], [\theta] = [0, 2\pi], \]
\[ [w_{ix}] = [-\infty, +\infty], [w_{iy}] = [-\infty, +\infty], \]
\[ [d_{ix}] = \text{obtained from the sensor}, \]
\[ [d_{iy}] = \text{obtained from the sensor}\}. \]

\[ \mathcal{C} = \left\{ \right. \]
\[ c_{w_{ix}} : d_{ix} \cos(\theta) + d_{iy} \sin(\theta) + x \]
\[ c_{w_{iy}} : -d_{ix} \sin(\theta) + d_{iy} \cos(\theta) + y \]
\[ c_{d_i} : \|d_i\|^2 = (x - w_{ix})^2 + (y - w_{iy})^2 \]
\[ c_{d_{i,j}} : \|d_i - d_j\|^2 = (w_{ix} - w_{jx})^2 + (w_{iy} - w_{jy})^2 \]
\[ c_G : \text{to be consistent with the map (using } C_G) \]
The Global Localization Algorithm

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The Global Localization Algorithm

Assumption

We assume \( u = (u_r, u_l) \) the knowledge of the moving of the robot, with \( u_r \) the speed of the right wheel and \( u_l \) the speed of the left wheel.
The Global Localization Algorithm

Prediction Equation

\[
f(p_k, u_k) = p_{k+1} = \begin{pmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{pmatrix} = \begin{pmatrix} x_k + \frac{u_{kr} + u_{kl}}{2} \cos(\theta_k) \\ y_k + \frac{u_{kr} + u_{kl}}{2} \sin(\theta_k) \\ \theta_k + \frac{u_{kr} - u_{kl}}{2} \end{pmatrix}.
\]

Retro-propagation Equation

\[
f^{-1}(p_{k+1}, u_k) = p_k = \begin{pmatrix} x_{k+1} - \frac{u_{kr} + u_{kl}}{2} \cos(\theta_{k+1} - \frac{u_{kr} - u_{kl}}{2}) \\ y_{k+1} - \frac{u_{kr} + u_{kl}}{2} \sin(\theta_{k+1} - \frac{u_{kr} - u_{kl}}{2}) \\ \theta_{k+1} - \frac{u_{kr} - u_{kl}}{2} \end{pmatrix}.
\]
Three Constraints

To be consistent with a data set and the map: The static localization.

Retro-propagation: \( p_{k-1} = f^{-1}(p_k, u_{k-1}) \),

Prediction: \( p_{k+1} = f(p_k, u_k) \),

with \( p_{k-1} \in \mathbb{P}_{k-1}, p_k \in \mathbb{P}_k, p_{k+1} \in \mathbb{P}_{k+1}, u_{k-1} \in [u_{k-1}] \) and \( u_k \in [u_k] \).
The Global Localization Algorithm

$\mathcal{P}_0$
The Global Localization Algorithm

$P_0$

Static localization
The Global Localization Algorithm

\[ f(p_0, u_0) \]

\[ P_0 \rightarrow P_1 \]
The Global Localization Algorithm

Static localization

\( P_0 \)  \hspace{2cm} \( P_1 \)
The Global Localization Algorithm

\[ f^{-1}(p_1, u_0) \]
The Global Localization Algorithm

\[ f(p_0, u_0) \]

\[ f^{-1}(p_1, u_0) \]
The Global Localization Algorithm

\[ f(p_1, u_1) \]
The Global Localization Algorithm

\[ P_0 \quad P_1 \quad P_2 \]

Static localization
The Global Localization Algorithm

\[ f(p_0, u_0) \xrightarrow{\text{Static localization}} P_0 \xrightarrow{f^{-1}(p_1, u_0)} P_1 \xrightarrow{f(p_1, u_1)} P_2 \xrightarrow{f^{-1}(p_2, u_1)} P_0 \]
The Global Localization Algorithm

\[ f(p_0, u_0) \rightarrow f(p_1, u_1) \rightarrow f(p_2, u_2) \rightarrow \ldots \]

Static localization

\[ f^{-1}(p_1, u_0) \rightarrow f^{-1}(p_2, u_1) \rightarrow f^{-1}(p_3, u_2) \]
The Simulator

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Simulation parameters

- A 1000 × 1000 cells occupancy grid map (10 × 10 meters).
- Each measurement has a bounded error of 10 centimetres and a max range $d_{max} = 3$ meters.
Demonstration
In this presentation we have seen that interval analysis could be used to solve the global localization problem.

This method uses a discrete map so the time processing depends on the size of the grid.

The simulation results are promising and this method can be efficient in a real context.
Conclusion

Future work

★ Experimental tests after the CAROTTE challenge.
★ The using of topological maps.
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Questions